1. SUMMATION OF SERIES (ARITHMETIC PROGRESSION)

Sum of first 'n' natural numbers:
$S = [n \times (n + 1)] / 2$

Sum of squares of first 'n' natural numbers:
$S = [n \times (n + 1) \times (2n + 1)] / 6$

Sum of cubes of first 'n' natural numbers:
$S = [(n \times (n + 1)) / 2]^2$

Sum of first 'n' odd numbers:
$S = n^2$

Sum of first 'n' even numbers:
$S = n \times (n + 1)$

2. DIVISORS AND FACTORS
For a number $N = a^p \times b^k \times c^r$ (where a, b, c are prime factors):

Total Number of Factors:
$(p + 1) \times (k + 1) \times (r + 1)$

Sum of All Factors:
$[(a^{p+1} - 1) / (a - 1)] \times [(b^{k+1} - 1) / (b - 1)] \times [(c^{r+1} - 1) / (c - 1)]$

Product of Factors:
$N^{(\text{Total Factors} / 2)}$

3. REMAINDER THEOREMS

Fermat’s Little Theorem:
If 'p' is a prime number, then $(a^{p-1} - 1)$ is divisible by p.
Or: $a^{p-1} \div p$ leaves Remainder = 1.

Euler’s Totient Theorem:
If HCF (a, n) = 1, then $a^{\Phi(n)} \div n$ leaves Remainder = 1.
(Where Φ (n) is Euler's totient count).

Dividend Rule:
Dividend = (Divisor × Quotient) + Remainder

4. DIVISIBILITY RULES (QUICK CHECK)
2^n: Last 'n' digits must be divisible by 2^n (e.g., $4 = 2^2$, check last 2 digits).
3: Sum of digits must be divisible by 3.
9: Sum of digits must be divisible by 9.

11: (Sum of digits at odd places) - (Sum of digits at even places) = 0 or 11k.
7, 11, 13: Difference between the last 3 digits and the remaining number must be divisible by 7, 11, or 13.

5. UNIT DIGIT CYCLICITY
0, 1, 5, 6: Cyclicity is 1 (Unit digit stays the same for any power).
4, 9: Cyclicity is 2.
$4^1 = 4, 4^2 = 6 \mid 9^1 = 9, 9^2 = 1$
2, 3, 7, 8: Cyclicity is 4.
(Divide the power by 4 and use the remainder as the new power).

6. MISCELLANEOUS ALGEBRAIC IDENTITIES
$(x^n - a^n)$ is always divisible by $(x - a)$ for all 'n'.
$(x^n - a^n)$ is divisible by $(x + a)$ only if 'n' is even.
$(x^n + a^n)$ is divisible by $(x + a)$ only if 'n' is odd.
Number of trailing zeros in n!:
Count of 5s = [n/5] + [n/25] + [n/125] + ...

Q1. Find the unit digit of $(6374)^{1793} \times (625)^{317} \times (341)^{491}$.
Solution: For unit digits, use cyclicity of 4.
$4^{odd}=4$. So, $(6374)^{1793} \rightarrow 4$.
5^n always ends in 5. So, $(625)^{317} \rightarrow 5$.
1^n always ends in 1. So, $(341)^{491} \rightarrow 1$.
Result: $4\times5\times1=20$. Unit digit is 0.

Q2. Find the number of zeros at the end of 100!+200!.
Solution: In addition, the number of zeros is determined by the smaller factorial.
Zeros in 100!: [100/5]+[100/25]=20+4=24.
Zeros in 200!: [200/5]+[200/25]+[200/125]=40+8+1=49.
Result: 24

Q3. Find the unit digit of 1!+2!+3!+...+100!.
Solution: From 5! onwards, the unit digit is always 0 (120,720,etc.).
We only sum 1!+2!+3!+4!=1+2+6+24=33.
Unit digit is 3.

Q4. Find the remainder when 2^{31} is divided by 5.
By Fermat's Theorem: 2^4 divided by 5 gives remainder 1.
$2^{31} = (2^4)^7 \times 2^3$.
Remainder $= (1)^7 \times 8 = 8$.
8 divided by 5 gives remainder 3.
Result: 3

Q5. Find the remainder of $(67^{67} + 67) \div 68$.
67 divided by 68 gives a negative remainder of -1.
Expression: $(-1)^{67} + (-1) = -1 - 1 = -2$.

Positive remainder: 68 - 2 = 66.
Result: 66

Q6. Find the remainder when 1234...99100 is divided by 16.
Divisibility rule for 16: Check the last 4 digits.
Last 4 digits are 9100.
9100 ÷ 16 = 568 with a remainder of 12.
Result: 12

Q7. If 789x531y is divisible by 72, find (5x - 3y).
Divisible by 8: Last 3 digits 31y must be divisible by 8. 312 ÷ 8 = 39, so y = 2.
Divisible by 9: Sum of digits (7+8+9+x+5+3+1+2) = 35+x.
35 + x must be divisible by 9. So x = 1.
Value: 5(1) - 3(2) = 5 - 6 = -1.
Result: -1

Q8. How many pairs (x, y) such that 34x5y is divisible by 36?
Must be divisible by 4 and 9.
Divisible by 4: 5y must be div by 4. So y = 2 or 6.
If y=2: 14+x must be div by 9, so x=4. Pair (4,2).
If y=6: 18+x must be div by 9, so x=0 or 9. Pairs (0,6), (9,6).
Result: 3 pairs

Q9. Find the number of even factors of 360.
$360 = 2^3 \times 3^2 \times 5^1$.
Number of even factors = (Power of 2) × (Power of 3 + 1) × (Power of 5 + 1).
Calculation: 3 × (2+1) × (1+1) = 3 × 3 × 2 = 18.
Result: 18

Q10. Find the sum of reciprocals of all factors of 360.
Formula: Sum of all factors ÷ Number itself.
Sum of factors = $(2^0+2^1+2^2+2^3)(3^0+3^1+3^2)(5^0+5^1)$ = 15 × 13 × 6 = 1170.
Reciprocal sum: 1170 ÷ 360 = 3.25.
Result: 3.25

Q11. Which is larger: $\sqrt[3]{4}$ or $\sqrt[4]{6}$?
Convert to 12th root: $4^{(1/3)} = 4^{(4/12)} = (4^4)^{(1/12)} = 256^{(1/12)}$
$6^{(1/4)} = 6^{(3/12)} = (6^3)^{(1/12)} = 216^{(1/12)}$.
Result: $\sqrt[3]{4}$

Q12. Value of √(12 + √(12 + √(12 + ... ∞))).
12 = 3 × 4. For positive sign (+), pick the larger consecutive factor.
Result: 4

Q13. Simplify: 1/(1×2) + 1/(2×3) + ... + 1/(99×100).
Telescoping sum: (1/1 - 1/2) + (1/2 - 1/3) + ... + (1/99 - 1/100).

Remaining terms: $1 - 1/100 = 99/100$.
Result: 0.99

Q14. If $x = 3 + 2\sqrt{2}$, find $x + 1/x$.
$1/x = 1 / (3 + 2\sqrt{2})$. Rationalize: $3 - 2\sqrt{2}$.
Sum: $(3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) = 6$.
Result: 6

Q15. A number divided by 899 leaves rem 63. If divided by 29, what is the rem?
29 is a factor of 899. Divide 63 by 29.
$63 = 29 \times 2 + 5$.
Result: 5

Q16. Numbers between 300 and 700 divisible by 5, 6, and 8?
Must be divisible by LCM(5, 6, 8) = 120.
Multiples: 360, 480, 600.
Result: 3

Q17. Value of $(0.111... + 0.222...) \times 3$.
$(1/9 + 2/9) \times 3 = 3/9 \times 3 = 1$.
Result: 1

Q18. $N \div 3$ rem 1. Quotient $\div$ 2 rem 1. $N \div 6$ rem?
Let Quotient $= 2k + 1$.
$N = 3(2k + 1) + 1 = 6k + 4$.
Result: 4

Q19. Largest power of 7 that divides 1000!.
$[1000/7] + [1000/49] + [1000/343] = 142 + 20 + 2 = 164$.
Result: 164

Q20. If $n^2 - 1$ is divisible by 8, then n is?
Test odd numbers: $3^2-1=8$, $5^2-1=24$.
Result: Any odd integer

Q21. Sum of 2-digit numbers leaving rem 3 when divided by 5.
AP: 13, 18, ... 98. $n = 18$.
Sum $= (18/2) \times (13 + 98) = 9 \times 111 = 999$.
Result: 999

Q22. If a, b are odd, which is even? (a+b, ab, ab+1).
Odd + Odd = Even.
Result: a+b and ab+1

Q23. HCF of $(2^{120} - 1)$ and $(2^{50} - 1)$.

HCF = 2^(HCF of 120, 50) - 1.
HCF(120, 50) = 10.
Result: $2^{10} - 1 = 1023$

Q24. A 6-digit number abcabc is always divisible by?
abcabc = abc × 1001.
1001 = 7 × 11 × 13.
Result: 7, 11, 13

Q25. Value of 999 (995/999) × 999.
$(999 + 995/999) \times 999 = 999^2 + 995$.
$(1000 - 1)^2 + 995 = 1{,}000{,}000 - 2000 + 1 + 995 = 998{,}996$.
Result: 998,996

Q26. If 48327*8 is divisible by 11, find *.
(4+3+7+8) - (8+2+x) = 22 - (10+x) = 12 - x.
12 - x = 11 => x = 1.
Result: 1

Q27. Square root of 7 + 4√3.
$(2 + \sqrt{3})^2 = 4 + 3 + 2(2)(\sqrt{3}) = 7 + 4\sqrt{3}$.
Result: 2 + √3

Q28. Multiples of both 3 and 4 between 1 and 200?
200 ÷ 12 = 16.
Result: 16

Q29. Sum of 3 consecutive primes is 173. Largest?
Primes: 53 + 59 + 61 = 173.
Result: 61

Q30. If sum of cubes 1 to 10 is 3025, find 4 + 32 + ... + 4000.
$4 \times (1^3 + 2^3 + ... + 10^3) = 4 \times 3025 = 12{,}100$.
Result: 12,100

Q31. Find the value of √(56 + √(56 + √(56 + ... ∞))).
Concept: Consecutive factors of x.
56 = 7 × 8.
Since the sign is (+), pick the larger factor.
Result: 8

Q32. Find the value of √(110 - √(110 - √(110 - ... ∞))).
Concept: Consecutive factors of x.
110 = 10 × 11.
Since the sign is (-), pick the smaller factor.
Result: 10

Q33. Find the value of $\sqrt{(13 \times \sqrt{(13 \times \sqrt{(13 \times \ldots \infty)})})}$.
Concept: Infinite multiplication of square roots.
Rule: $\sqrt{(x \times \sqrt{(x \ldots \infty)})} = x$.
Result: 13

Q34. Find the value of $\sqrt{(3 \times \sqrt{(3 \times \sqrt{(3)})})}$.
Concept: Finite multiplication of square roots.
Formula: $x^{[(2^n - 1) / 2^n]}$. Here $x = 3$, $n = 3$.
Calc: $3^{[(2^3 - 1) / 2^3]} = 3^{[(8 - 1) / 8]}$.
Result: $3^{7/8}$

Q35. Find the value of $\sqrt[3]{(16 \times \sqrt[3]{(16 \times \sqrt[3]{(16 \times \ldots \infty)})})}$.
Concept: Infinite multiplication of n-th roots.
Rule: Value = (n-1)th root of x.
Calc: $\sqrt[2]{16} = 4$.
Result: 4

Q36. Find the value of $\sqrt{(17 + \sqrt{(17 + \sqrt{(17 + \ldots \infty)})})}$.
Concept: Non-consecutive factors formula.
Formula: $[\sqrt{(4x + 1)} + 1] / 2$.
Calc: $[\sqrt{(4 \times 17 + 1)} + 1] / 2 = [\sqrt{69} + 1] / 2$.
Result: $(\sqrt{69} + 1) / 2$

Q37. Find the value of $\sqrt{(7 - \sqrt{(7 - \sqrt{(7 - \ldots \infty)})})}$.
Concept: Non-consecutive factors (Negative).
Formula: $[\sqrt{(4x + 1)} - 1] / 2$.
Calc: $[\sqrt{(4 \times 7 + 1)} - 1] / 2 = [\sqrt{29} - 1] / 2$.
Result: $(\sqrt{29} - 1) / 2$

Q38. Find the value of $\sqrt{(4 + \sqrt{(4 - \sqrt{(4 + \sqrt{(4 - \ldots \infty)})})})}$.
Concept: Alternating signs starting with (+).
Formula: $[\sqrt{(4x - 3)} + 1] / 2$.
Calc: $[\sqrt{(4 \times 4 - 3)} + 1] / 2 = [\sqrt{13} + 1] / 2$.
Result: $(\sqrt{13} + 1) / 2$

Q39. Find the value of $\sqrt{(8 - \sqrt{(8 + \sqrt{(8 - \sqrt{(8 + \ldots \infty)})})})}$.
Concept: Alternating signs starting with (-).
Formula: $[\sqrt{(4x - 3)} - 1] / 2$.
Calc: $[\sqrt{(4 \times 8 - 3)} - 1] / 2 = [\sqrt{29} - 1] / 2$.
Result: $(\sqrt{29} - 1) / 2$

Q40. Find the value of $\sqrt{(125 \div \sqrt{(125 \div \sqrt{(125 \div \ldots \infty)})})}$.
Concept: Infinite division of square roots.
Rule: Value = $\sqrt[3]{x}$.
Calc: $\sqrt[3]{125} = 5$.
Result: 5

Q41. Find the value of $\sqrt[4]{(27 \times \sqrt[4]{(27 \times \sqrt[4]{(27 \times \ldots \infty)})})}$.
Concept: Infinite multiplication of 4th roots.
Rule: (n-1)th root of x.
Calc: $\sqrt[3]{27} = 3$.
Result: 3

Q42. If $\sqrt{(x\sqrt{(x\sqrt{(x)})})} = 5$, find x.
Calc: $x^{[(2^3-1)/2^3]} = 5 \Rightarrow x^{7/8} = 5$.
$x = 5^{8/7}$.
Result: $5^{8/7}$

Q43. Find the value of $\sqrt{(2 + \sqrt{(2 + \sqrt{(2 + \ldots \infty)})})}$.
Calc: $2 = 1 \times 2$. Larger factor is 2.
Result: 2

Q44. Find the value of $\sqrt{(20 - \sqrt{(20 - \sqrt{(20 - \ldots \infty)})})}$.
Calc: $20 = 4 \times 5$. Smaller factor is 4.
Result: 4

Q45. Simplify: $(\sqrt{(2 \times \sqrt{(2 \times \sqrt{(2 \times \sqrt{(2)})})})}) \div (\sqrt{(2 \times \sqrt{(2\ldots \infty)})})$.
Numerator: $2^{15/16}$.
Denominator: 2.
Calc: $2^{15/16} \div 2^1 = 2^{(15/16 - 1)} = 2^{-1/16}$.
Result: $1 / (2^{1/16})$

Q51. Find the sum of all 3-digit numbers that are exactly divisible by 11.
Concept: Sum of an A.P.
First 3-digit multiple of 11 (a) = 110.
Last 3-digit multiple of 11 (l) = 990.
Common difference (d) = 11.
Find number of terms (n): n = [(l - a) / d] + 1
n = [(990 - 110) / 11] + 1 = [880 / 11] + 1 = 81.
Sum = (n / 2) × (a + l)
Sum = (81 / 2) × (110 + 990) = (81 / 2) × 1100 = 81 × 550.
Calculation: 81 × 550 = 44,550.
Result: 44,550

Q52. Find the sum of the first 20 terms of the series: 1 + 2 + 3 - 4 + 5 + 6 + 7 - 8 + ...
Concept: Pattern-based Summation.
Group every 4 terms: (1 + 2 + 3 - 4) = 2.
Group 2: (5 + 6 + 7 - 8) = 10.
Group 3: (9 + 10 + 11 - 12) = 18.
This forms a new A.P. of groups: 2, 10, 18 ...
Since there are 20 terms, and we grouped by 4, there are 20/4 = 5 groups.
For this A.P.: A = 2, D = 8, N = 5.
Sum = (N / 2) × [2A + (N - 1)D]

Sum = $(5 / 2) \times [2(2) + (4)8] = (5 / 2) \times [4 + 32] = (5 / 2) \times 36 = 5 \times 18$.
Result: 90

Q53. The sum of three consecutive terms in an A.P. is 27 and their product is 504. Find the terms.
Concept: Selection of terms in A.P.
Let terms be $(a - d), a, (a + d)$.
Sum: $(a - d) + a + (a + d) = 27 \Rightarrow 3a = 27 \Rightarrow a = 9$.
Product: $(a - d) \times a \times (a + d) = 504$
$(9 - d) \times 9 \times (9 + d) = 504$
$(81 - d^2) = 504 / 9 = 56$.
$d^2 = 81 - 56 = 25 \Rightarrow d = 5$.
Terms: $(9 - 5), 9, (9 + 5) = 4, 9, 14$.
Result: 4, 9, 14

Q54. Find the sum of the series: $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 \ldots$ up to 100 terms.
Concept: Algebraic Identity $a^2 - b^2 = (a - b)(a + b)$.
Group in pairs: $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) \ldots + (99^2 - 100^2)$
Apply identity: $(1-2)(1+2) + (3-4)(3+4) + (5-6)(5+6) \ldots + (99-100)(99+100)$
Factor out (-1): $-1 \times (3 + 7 + 11 + \ldots + 199)$.
This is an A.P. with $a = 3$, $l = 199$, $n = 50$.
Sum = $-1 \times [(50 / 2) \times (3 + 199)] = -1 \times [25 \times 202]$.
Result: -5050

Q55. Find the sum of all natural numbers between 100 and 400 which are divisible by both 4 and 6.
Concept: LCM and A.P.
Numbers divisible by both 4 and 6 must be divisible by LCM(4, 6) = 12.
First number after 100 divisible by 12: 108.
Last number before 400 divisible by 12: 396.
Find n: $n = [(396 - 108) / 12] + 1 = [288 / 12] + 1 = 24 + 1 = 25$.
Sum = $(25 / 2) \times (108 + 396) = (25 / 2) \times 504 = 25 \times 252$.
Calculation: $25 \times 252 = 6{,}300$.
Result: 6,300